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Direct Boundary Element Method for Calculation of Pressure Distribution over the Boundary of a Symmetric Aerofoil

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Abstract: In this paper, the direct boundary element method is being used to give solution for surface as well as ground water bodies which are major nutrition fluids. To calculate the pressure distribution over the boundary of a symmetric aerofoil for which the analytical solution is available to check the accuracy of the method, the computed flow pressure are compared with the analytical solution for the flow over the boundary of a symmetric aerofoil.

Key words: Direct boundary element method, pressure distribution, symmetric aerofoil

INTRODUCTION

The boundary element method is frequently used to determine the pressure distribution over the boundary of water body which is a major nutrition source. In addition to problem related to water resources this method can be used to evaluate pressure distribution over the boundary of a symmetric aerofoil.

The boundary element method is a method for solving partial differential equations and it is only active when the physical problem can be expressed in a PDEs. In engineering and physical sciences, theboundary element method has steadily developed to become one of the rare extensively used numerical techniques to solve boundary value problems, in the course of last few eras.

It is derived through the discretization of an integral equation which is mathematically equals to the original partial differential equation. The benefits of the boundary element method are that only the boundary of the domain of the PDE necessitates the discretization to produce a surface or boundary mesh. Boundary element method also reduces the dimension of the problem by one e.g. an equation in three-dimensional region is transformed into one over its surface and the equation having infinite domain is reduced to an equation over the (finite) boundary. This reduction of dimension leads to smaller linear systems, less computer memory requirements and more efficient computation.

BEM is sub-divided into two groups namely Direct Boundary Element method and Indirect Boundary Element method. The Direct method is based on the matrix factorization. While the Iterative method uses the distribution of the singularities over the surface of the body to compute the solution of the integral equation.

There are two approaches to formulate the equation of direct boundary element method, first is based on Green's theorem as used by Lamb (1932), Milne-Thomson (1967), Kellogg (1929), Ramsey (1942) and Shah (2008) and the other is a particular case of the weighted residual methods as used by Brebbia (1978); Brebbia and Walker (1980).

In 1960, with the invention of the computer and the development in the first high level programming language, the approximated numerical solution of boundary value problems became possible (Watson, 2003). Boundary element method initiated within the Department of civil engineering at Southampton University, U.K.

In 1967-1973, Hess and Smith (1967); Hess (1973) also used indirect boundary element method for solving higher order potential flow problems. In 1975, Morino *et al*. (1975) used the direct boundary element method for the calculations of flow field.

The period of 1980 to 1993 was the time of betterment and development of the method.In the past, these methods were developed in all zones such as stress analysis, heat transfer and electromagnetic theory, potential theory, fracture mechanics, fluid mechanics, elasticity, elasto-statics and elasto-dynamics, biological and biomedical problems, etc. (Shah, 1985; Hess, 1990; Banerjee and Morino, 1990; Morino, 1993). In 2000, Kohr (2000) used the direct BEM for a mobility problem.

Luminita (2008) and Shah (1985) developed an indirect boundary element method for the flow field calculations around arbitrary bodies.

In 2008-2010, Mushtaq *et al*. (2008, 2009); Mushtaq and Shah (2010c,d); Mushtaq (2011) used boundary element method for the calculations of flow field and also they presented a comparison of direct and indirect boundary element method by using different approaches. In 2010, Mushtaq (2010b) presented the advantages and

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disadvantages of using boundary element methods and also they presented the developments and applications of the method (Mushtaq *et al.*, 2010a).

In 2011, Mushtaq and Muhammad used boundary element methods for compressible and incompressible flows in their Ph.D. thesis.

In the past the calculation of inviscid flow field around arbitrary bodies has always been obtained using an indirect boundary element method. As declared above that only Morino *et al.* (1993) use the direct boundary element method to calculate the flow field calculations around aircraft. Thus there is a necessity to apply the direct boundary element method to calculate the ideal flow field around arbitrary body i.e. symmetric aerofoil and compare the results with analytic solutions.

MATERIALS AND METHODS

Flow past a symmetric aerofoil: For an exterior compressible flow problem, consider that the uniform stream with velocity U_1 in the positive x_1 - axis direction passes over a symmetric aerofoil as shown in Fig. 1.

Chow (1979) and Mushtaq and Shah (2010); Mushtaq (2011) given the magnitude of the exact velocity distribution over the boundary of a symmetric aerofoil as:

$$V_1 = U_1 \left| \frac{1-\left(\frac{r_1}{z_1 - a_1}\right)^2}{1-\left(\frac{b_1}{z_1}\right)} \right| \quad (1)$$

where, r_1 is the radius of the circular cylinder (C.C) and b_1 is the constant of Joukowski transformation and $a_1 = b_1 - r_1 = x_1$-coordinate is the center of the C.C.

Hence in rectangular coordinates, Eq. (1) can be written as:

$$V_1 = U_1 \sqrt{\frac{[\{(x_1-a_1)^2+y_1^2\}^2 - r^2\{(x_1-a_1)^2-y_1^2\}]^2 + 4r^4 y_1 (x_1-a_1)^2}{[(x_1-a_1)^2+y_1^2]^2}} \times \sqrt{\frac{[(x_1^2+y_1^2)^2 - b_1^2(x_1^2-y_1^2)]^2 + 4b_1^4 x_1^2 y_1^2}{(x_1^2+y_1^2)^2 - 2b_1^2(x_1^2-y_1^2) + b_1^4}} \quad (2)$$

Boundary condition of aerofoil: Now the boundary condition [See Mushtaq *et al.* (2008), Mushtaq and Shah (2010c,d), Mushtaq (2011)] of symmetric aerofoil is:

$$\overline{V}_1 . \hat{n} = 0 \quad (3)$$

where, $\hat{n}$ is the unit normal to the boundary of the symmetric aerofoil.

Since the motion is irrotational:

$$\overline{V}_1 = -\nabla\phi_1 \quad (4)$$

where, ϕ_1 is the total velocity potential. Thus from equations (3) and (4), we get:

$$-\nabla\phi_1 \bullet \hat{n} = 0 \quad (5)$$

Or:

$$\frac{\partial\phi_1}{\partial n} = 0 \quad (6)$$

Now the total velocity potential ϕ_1 can be written as:

$$\phi_1 = \phi_{u.s} + \phi_{s.a} \quad (7)$$

Or:

$$\frac{\partial\phi_1}{\partial n} = \frac{\partial\phi_{u.s}}{\partial n} + \frac{\partial\phi_{s.a}}{\partial n} \quad (8)$$

Hence from equations (6) and (8), we get:

$$\frac{\partial\phi_{u.s}}{\partial n} + \frac{\partial\phi_{s.a}}{\partial n} = 0$$
$$\frac{\partial\phi_{s.a}}{\partial n} = -\frac{\partial\phi_{u.s}}{\partial n} \quad (9)$$

But the velocity potential of the uniform stream, given in Milne-Thomson (1967), Shah (2008), is:

$$\phi_{u.s} = -U_1 x \quad (10)$$

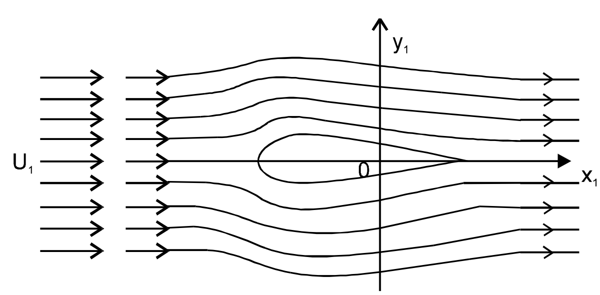

Fig. 1: Flow past a Symmetric aerofoil

Then:

$$\frac{\partial \phi_{u.s}}{\partial n} = -U_1 \frac{\partial x}{\partial n} = -U_1\left(\hat{n}\cdot\hat{i}\right) \quad (11)$$

Hence from equations (9) and (11), we get:

$$\frac{\partial \phi_{u.s}}{\partial n} = U_1\left(\hat{n}\cdot\hat{i}\right) \quad (12)$$

In general the outward drawn unit normal n̂ to any vector $\vec{p}$:

$$= (X_b - X_a)\hat{i} + (Y_b - Y_a)\hat{J}$$

is given by:

$$\hat{n} = \frac{-\left(Y_b - Y_a\right)\hat{i} + \left(X_b - X_a\right)\hat{J}}{\sqrt{\left(X_b - X_a\right)^2 + \left(Y_b - Y_a\right)^2}}$$

Thus:

$$\hat{n}\cdot\hat{i} = \frac{-\left(Y_b - Y_a\right)}{\sqrt{\left(X_b - X_a\right)^2 + \left(Y_b - Y_a\right)^2}} \quad (13)$$

From equations (12) and (13), we get:

$$\frac{\partial \phi_{s.a}}{\partial n} = U_1\left(\frac{-\left(Y_b - Y_a\right)}{\sqrt{\left(X_b - X_a\right)^2 + \left(Y_b - Y_a\right)^2}}\right) \quad (14)$$

This is the boundary condition which is essentially being fulfilled over the boundary of a symmetric aerofoil.

Discritization of direct boundary element equation: Two-dimensional interior or exterior ideal flow problems are solvable by Boundary element methods. Now the coordinates of the extreme points of the boundary elements for the discretization of the boundary of the symmetric aerofoil are given in following manner. Divide the boundary of the circular cylinder into P elements in the clockwise direction by using the formula [see Mushtaq (2011), Muhammad (2011):

$$\theta_j = \left[(P+3) - 2_j\right]\frac{\pi}{P} \qquad j = 1, 2, \ldots, P \quad (15)$$

Then the extreme points of these P elements of circular cylinder are given by:

$$\left.\begin{aligned} \xi_j &= -a + r\ \text{csin}\theta_1 \\ \eta_j &= r\sin\theta_1 \end{aligned}\right\} \quad (16)$$

Now by using Joukowski's transformation, the extreme points of the symmetric aerofoil are:

$$z_j = \zeta_j + \frac{b^2}{\zeta_j} \quad (17)$$

Now if $\zeta_j = \xi_j + i\eta_j$ and $z_j = x_j + iy_j$ then Eq. (17) becomes:

$$x_j + iy_j = \xi_j + i\eta_j + \frac{b^2}{\xi_j + i\eta_j}$$

$$x_j + iy_j = \xi_j + i\eta_j + \frac{b^2\left(\xi_j - i\eta_j\right)}{\xi_j^2 + \eta_j^2}$$

$$x_j + iy_j = \left(\xi_j + \frac{b^2\xi_j}{\xi_j^2 + \eta_j^2}\right) + i\left(\eta_j \frac{b^2\eta_j}{\xi_j^2 + \eta_j^2}\right)$$

$$x_j + iy_j = \left(1 + \frac{b^2}{\xi_j^2 + \eta_j^2}\right)\xi_j + i\left(1 - \frac{b^2}{\xi_j^2 + \eta_j^2}\right)\eta_j$$

Comparing real and imaginary parts:

$$\left.\begin{aligned} xj &= \left(1 + \frac{b^2}{\xi_j^2 + \eta_j^2}\right)\xi_j \\ y_j &= \left(1 - \frac{b^2}{\xi_j^2 + \eta_j^2}\right)\eta_j \end{aligned}\right\} \quad j = 1, 2, \ldots, P \quad (18)$$

Matrix formulation: Since the equation of direct boundary element method is:

$$c_Q\phi_Q + \phi\infty = \frac{1}{4\pi}\iint_{\chi}\frac{1}{r}\frac{\partial\phi_1}{\partial n}d\chi - \frac{1}{4\pi}\iint_{\chi-Q}\phi_1\frac{\partial}{\partial n}\left(\frac{1}{r}\right)d\chi \quad (19)$$

This equation cannot be solved analytically. So rearrangement is made to use numerical techniques. Now in this equation, integrals are involved which are to be evaluated at the surface χ of the body. Then surface of the body can be discretized into finite number of boundary elements. When surface of the body is divided into finite number of elements then integrals takes the form of summation of the integrals taken over all the elements. Thus these integrals are reduced to algebraic equations, which are easy to solve by using numerical techniques.

Let the surface χ of the symmetric aerofoil be divided into finite number of boundary elements. Then Eq. (19) can be written as:

$$c_Q\phi_Q + \phi\infty + \sum_{i=Q}^{P}\iint_{\chi i-Q}\phi_i\frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right)d\chi = \sum_{i=1}^{P}\iint_{\chi i}\frac{1}{4\pi r}\frac{\partial\phi_i}{\partial n}d\chi \quad (20)$$

where, χ_i-Q is the surface area of the element excluding the point Q.

For constant element approach, the values of $\partial\phi/\partial n$ and ϕ are assumed to be constant on each element and will be equal to the mid-node values of the elements. Now the coordinates of the middle node of each boundary element are given by:

$$\left.\begin{aligned} x_M &= \frac{x_j + x_{j+1}}{2} \\ y_M &= \frac{y_j + y_{j+1}}{2} \end{aligned}\right\} \quad j,\ M = 1, 2, \ldots\ldots, n \qquad (21)$$

Thus the numbers of nodes are equal to the number of elements in this case.

Since $\partial\phi/\partial n$ and ϕ are constant at each point, so:

$$c_Q\phi_Q + \phi\infty + \left(\sum_{i=Q}^{P}\iint_{\chi_i - Q}\frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right)d\chi\right)\phi_i = \left(\sum_{i=1}^{P}\iint_{\chi_i}\frac{1}{4\pi r}d\chi\right)\frac{\partial\phi_i}{\partial n} \qquad (22)$$

Now denoting the integral on L.H.S of equation (22) by $\widehat{W}_{Qi}$ and on R.H.S by T_{qi}:

$$c_Q\phi_Q + \phi\infty + \sum_{i=Q}^{P}\widehat{W}_{Qi}\ \phi_i = \sum_{i=1}^{P}T_{Qi}\frac{\partial\phi_i}{\partial n} \qquad (23)$$

Let:

$$W_{Qi} = \begin{cases} \widehat{W}_{Qi} & \text{when } Q \neq i \\ \widehat{W}_{Qi} + c_Q & \text{when } Q = 1 \end{cases} \qquad (24)$$

Then:

$$\phi_\infty + \sum_{i=1}^{P}W_{Qi}\ \phi_i = \sum_{i=1}^{P}T_{Qi}\frac{\partial\phi_i}{\partial n} \qquad (25)$$

This can take the form:

$$[W]\{\underset{\sim}{X}\} = [T]\{\underset{\sim}{Y}\} \qquad (26)$$

With the assumption that the values of P or P are given as boundary conditions then this equation becomes:

$$[A]\{\underset{\sim}{X}\} = [B] \qquad (27)$$

This equation represents a set of P linear equations in P unknowns. Now this system of equations can be solved numerically. From here, the ϕ can be found and then velocity and pressure at symmetric aerofoil can be calculated.

The velocity in the middle of two nodes on the boundary, can then be estimated by using the formula:

$$\overline{V_1} = \frac{\phi_{J+1} - \phi_J}{\text{Length from node j to } j+1} \qquad (28)$$

Once the velocity at a point becomes known. The pressure coefficients at that point can be calculation according to:

$$C_p = 1 - \left(\frac{V_1}{U}\right)^2 = 1 - V_1^2 \quad (\text{Take } U = 1) \qquad (29)$$

RESULTS AND DISCUSSION

An attempt has been made to calculate the pressure distribution through computer using FORTRAN language over the boundary of a symmetric Aerofoil. The data generated using this method with constant variation is presented in the following tables and compared with the analytical results.

Chow (1979) adopted analytical method to solve the problems related to pressure distribution around aerofoils. In the past, Mushtaq and Shah (2010) calculated the compressible flow using DBEM and

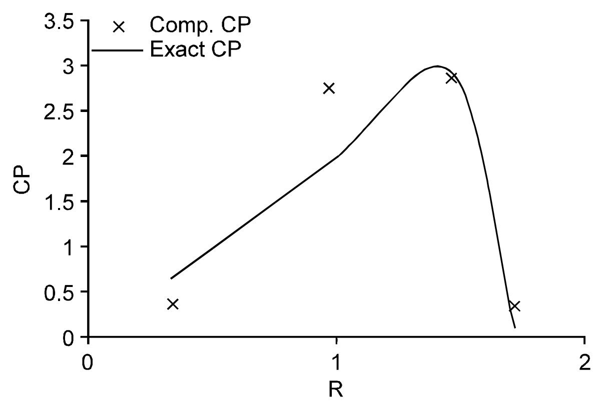

Fig. 2: Comparison of computed and analytical pressure distributions over the surface of symmetric aerofoil using 8 boundary elements with constant variation

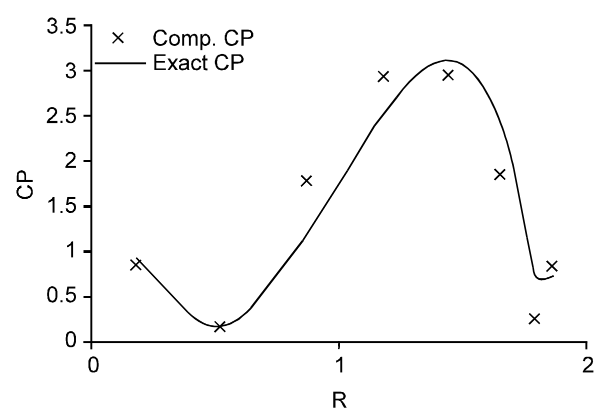

Fig. 3: Comparison of computed and analytical pressure distributions over the surface of symmetric aerofoil using 16 boundary elements with constant variation

Table 1: Comparison of the analytical and computed pressure distributions over the boundary of the symmetric aerofoil for 8 boundary elements with constant variation

Element	XM	YM	R	Computed CP	Exact CP
1	-1.69	0.33	1.72	0.34414E+00	0.11598E+00
2	-1.23	0.78	1.46	0.28119E+01	0.29354E+01
3	-0.58	0.78	0.97	0.27471E+01	0.19228E+01
4	-0.11	0.32	0.34	0.36849E+00	0.66481E+00
5	-0.11	-0.32	0.34	0.36849E+00	0.66481E+00
6	-0.58	-0.78	0.97	0.27471E+01	0.19228E+01
7	-1.23	-0.78	1.46	0.28119E+01	0.29354E+01
8	-1.69	-0.33	1.72	0.34415E+00	0.11599E+00

Table 2: Comparison of the analytical and computed pressure distributions over the boundary of the symmetric aerofoil for 16 boundary elements with constant variation

Element	XM	YM	R	Computed CP	Exact CP
1	-1.85	0.19	1.86	0.84287E+00	0.72867E+00
2	-1.71	0.53	1.79	0.27353E+00	0.74029E+00
3	-1.44	0.80	1.65	0.18487E+01	0.24246E+01
4	-1.09	0.94	1.44	0.29532E+01	0.31159E+01
5	-0.72	0.94	1.18	0.29306E+01	0.25422E+01
6	-0.37	0.79	0.87	0.17824E+01	0.12043E+01
7	-0.10	0.51	0.52	0.16062E+00	.16742E+00
8	0.06	0.17	0.18	0.84734E+00	0.92862E+00
9	0.06	-0.17	0.18	0.84734E+00	0.92862E+00
10	-0.10	-0.51	0.52	0.16062E+00	0.16742E+00
11	-0.37	-0.79	0.87	0.17824E+01	0.12043E+01
12	-0.72	-0.94	1.18	0.29306E+01	0.25422E+01
13	-1.09	-0.94	1.44	0.29532E+01	0.31159E+01
14	-1.44	-0.80	1.65	0.18487E+01	0.24246E+01
15	-1.71	-0.53	1.79	0.27353E+00	0.74029E+00
16	-1.85	-0.19	1.86	0.84288E+00	0.72867E+00

Table 3: Comparison og the analytical and computed pressure distributions over the boundary of the symmetric aerofoil for 32 boundary elements with constant variation

Element	XM	YM	R	Computed CP	Exact CP
1	-1.89	0.10	1.89	0.96111E+00	0.89468E+00
2	-1.85	0.29	1.88	0.65890E+00	0.46363E+00
3	-1.78	0.47	1.84	0.10074E+00	0.29329E+00
4	-1.67	0.63	1.78	0.62792E+00	0.12001E+01
5	-1.53	0.76	1.71	0.14155E+01	0.20627E+01
6	-1.37	0.87	1.62	0.21412E+01	0.27189E+01
7	-1.19	0.94	1.52	0.26935E+01	0.30683E+01
8	-1.00	0.98	1.40	0.29873E+01	0.30805E+01
9	-0.81	0.98	1.27	0.29766E+01	0.27847E+01
10	-0.62	0.94	1.12	0.26619E+01	0.22509E+01
11	-0.44	0.86	0.97	0.20901E+01	0.15700E+01
12	-0.28	0.75	0.80	0.13473E+01	0.83729E+00
13	-0.14	0.61	0.63	0.54622E+00	0.14178E+00
14	-0.03	0.44	0.44	0.18990E+00	0.43919E+00
15	0.06	0.25	0.26	0.72909E+00	0.83372E+00
16	0.13	0.8	0.15	0.94995E+00	0.86991E+00
17	0.13	-0.08	0.15	0.94995E+00	0.86991E+00
18	0.06	-0.25	0.26	0.72910E+00	0.83372E+00
19	-0.03	-0.44	0.44	0.18990E+00	0.43919E+00
20	-0.14	-0.61	0.63	0.54621E+00	0.14178E+00
21	-0.28	-0.75	0.80	0.13473E+01	0.83729E+00
22	-0.44	-0.86	0.97	0.20901E+01	0.15700E+01
23	-0.62	-0.94	1.12	0.26619E+01	0.22509E+01
24	-0.81	-0.98	1.27	0.29766E+01	0.27847E+01
25	-1.00	-0.98	1.40	0.29873E+01	0.30805E+01
26	-1.19	-0.94	1.52	0.26935E+01	0.30683E+01
27	-1.37	-0.87	1.62	0.21412E+01	0.27189E+01
28	-1.53	-0.76	1.71	0.14155E+01	0.20627E+01
29	-1.67	-0.63	1.78	0.62791E+00	0.12001E+01
30	-1.78	-0.47	1.84	0.10075E+00	0.29329E+00
31	-1.85	-0.29	1.88	0.65889E+00	0.46363E+00
32	-1.89	-0.10	1.89	0.96111E+00	0.89469E+00

Table 4: Comparison og the analytical and computed pressure distributions over the boundary of the symmetric aerofoil for 64 boundary elements with constant variation

Element	XM	YM	R	Computed CP	Exact CP
1	-1.90	0.05	1.90	0.99029E+00	0.93703E+00
2	-1.89	0.15	1.90	0.91326E+00	0.82486E+00
3	-1.87	0.24	1.89	0.76218E+00	0.60762E+00
4	-1.84	0.34	1.87	0.54282E+00	0.29891E+00
5	-1.81	0.43	1.86	0.26379E+00	0.82443E-01
6	-1.76	0.51	1.83	0.64200E-01	0.51364E+00
7	-1.71	0.59	1.81	0.42850E+00	0.97009E+00
8	-1.64	0.67	1.77	0.81522E+00	0.14271E+01
9	-1.58	0.74	1.74	0.12088E+01	0.18608E+01
10	-1.50	0.80	1.70	0.15947E+01	0.22512E+01
11	-1.42	0.85	1.65	0.19577E+01	0.25808E+01
12	-1.33	0.90	1.61	0.22833E+01	0.28373E+01
13	-1.24	0.94	1.55	0.25596E+01	0.30124E+01
14	-1.15	0.96	1.50	0.27753E+01	0.31027E+01
15	-1.05	0.98	1.44	0.29224E+01	0.31083E+01
16	-0.95	0.99	1.38	0.29944E+01	0.30333E+01
17	-0.86	0.99	1.31	0.29892E+01	0.28841E+01
18	-0.76	0.98	1.24	0.29066E+01	0.26703E+01
19	-0.66	0.96	1.17	0.27493E+01	0.24018E+01
20	-0.57	0.93	1.09	0.25235E+01	0.20906E+01
21	-0.48	0.89	1.01	0.22378E+01	0.17486E+01
22	-0.39	0.85	0.93	0.19033E+01	0.13879E+01
23	-0.31	0.79	0.85	0.15322E+01	0.10204E+01
24	-0.23	0.73	0.76	0.11392E+01	0.65663E+00
25	-0.16	0.66	0.68	0.73950E+00	0.30782E+00
26	-0.10	0.58	0.59	0.34862E+00	0.16967E-01
27	-0.05	0.49	0.50	0.17939E-01	0.30853E+00
28	0.00	0.40	0.40	0.34507E+00	0.55835E+00
29	0.04	0.31	0.31	0.61821E+00	0.75718E+00
30	0.08	0.20	0.22	0.82252E+00	0.89208E+00
31	0.12	0.10	0.16	0.93804E+00	0.93350E+00
32	0.16	0.03	0.17	0.72547E+00	0.83482E+00
33	0.16	-0.03	0.17	0.72547E+00	0.83482E+00
34	0.12	-0.10	0.16	0.93804E+00	0.93350E+00
35	0.08	-0.20	0.22	0.82252E+00	0.89208E+00
36	0.04	-0.31	0.31	0.61821E+00	0.75718E+00
37	0.00	-0.40	0.40	0.34507E+00	0.55835E+00
38	-0.05	-0.49	0.50	0.17939E-01	0.30853E+00
39	-0.10	-0.58	0.59	0.34862E+00	0.16967E-01
40	-0.16	-0.66	0.68	0.73950E+00	0.30782E+00
41	-0.23	-0.73	0.76	0.11392E+01	0.65663E+00
42	-0.31	-0.79	0.85	0.15322E+01	0.10204E+01
43	-0.39	-0.85	0.93	0.19033E+01	0.13879E+01
44	-0.48	-0.89	1.01	0.22378E+01	0.17486E+01
45	-0.57	-0.93	1.09	0.25235E+01	0.20906E+01
46	-0.66	-0.96	1.17	0.27493E+01	0.24018E+01
47	-0.76	-0.98	1.24	0.29066E+01	0.26703E+01
48	-0.86	-0.99	1.31	0.29892E+01	0.28841E+01
49	-0.95	-0.99	1.38	0.29944E+01	0.30333E+01
50	-1.05	-0.98	1.44	0.29224E+01	0.31083E+01
51	-1.15	-0.96	1.50	0.27753E+01	0.31027E+01
52	-1.24	-0.94	1.55	0.25596E+01	0.30124E+01
53	-1.33	-0.90	1.61	0.22833E+01	0.28373E+01
54	-1.42	-0.85	1.65	0.19577E+01	0.25808E+01
55	-1.50	-0.80	1.70	0.15947E+01	0.22512E+01
56	-1.58	-0.74	1.74	0.12088E+01	0.18608E+01
57	-1.64	-0.67	1.77	0.81522E+00	0.14271E+01
58	-1.71	-0.59	1.81	0.42850E+00	0.97009E+00
59	-1.76	-0.51	1.83	0.64200E-01	0.51364E+00
60	-1.81	-0.43	1.86	0.26379E+00	0.82443E-01
61	-1.84	-0.34	1.87	0.54282E+00	0.29891E+00
62	-1.87	-0.24	1.89	0.76218E+00	0.60762E+00
63	-1.89	-0.15	1.90	0.91326E+00	0.82486E+00
64	-1.90	-0.05	1.90	0.99029E+00	0.93703E+00

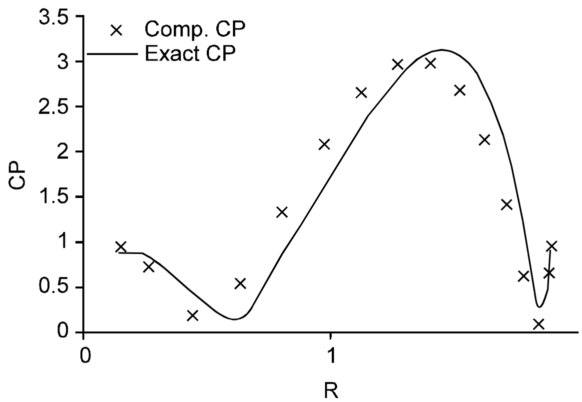

Fig. 4: Comparison of computed and analytical pressure distributions over the surface of symmetric aerofoil using 32 boundary elements with constant variation

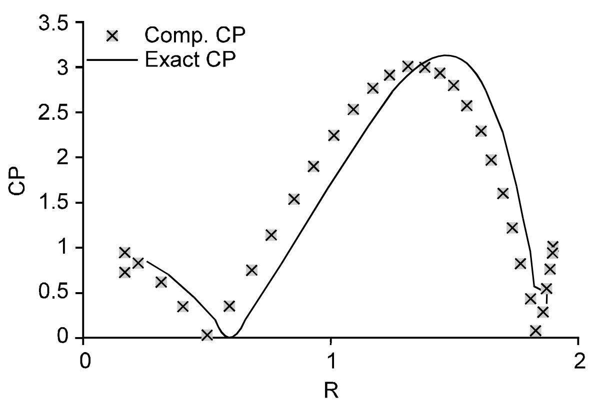

Fig. 5: Comparison of computed and analytical pressure distributions over the surface of symmetric aerofoil using 64 boundary elements with constant variation

compared with the analtyical results by Chow (1979). In this research paper, we are applying the same technique for Ideal flow and calculated the pressure distribution over the boundary of symmetric aerofoil. We see that the results for calculating the pressure distribution are in good agreement with the analytical results calculated by Chow (1979). It is attributed that this numerical technique is low cost and time saving and efficient compared to other numerical techniques (i.e., Domain techniques). Therefore, this study highlights the accuracy of computer generated solutions with various fluid flow problems related to water bodies being the main nutrition source.

Conclusion and recommendations: A direct boundary element method has been used for the calculation of ideal flow around two-dimensional body. The computed flow pressures obtained using this method is compared with the analytical solutions for flows over the boundary of a symmetric aerofoil. It is found that the results obtained with this method for the flow field calculations are in good agreement with the analytical results for the body under consideration.

Thus the direct boundary element method is being suggested to deal the fluid flow problems such as tides, turbulent or laminar thrust related to the submarines and other shipping vehicles.

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